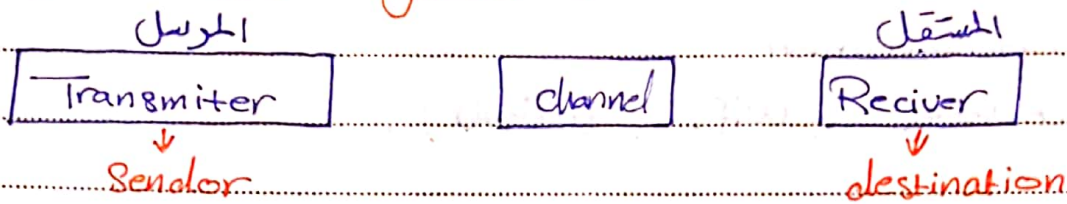


Communication System :-



* Communication channel :-

medium between a transmitter and its destination.

1) Wireline channel :-

Cables are normally used to connect a transmitter and a destination.

Ex : * Coaxial Cable.

* fiber optic is normally used since it has a huge Bandwidth.

2) Wireless channel :-

antennas are used for transmission and reception.

Application :-

on Wireline channel :-

* Broadband internet service.

* Fixed phone service.

on wireless channel :-

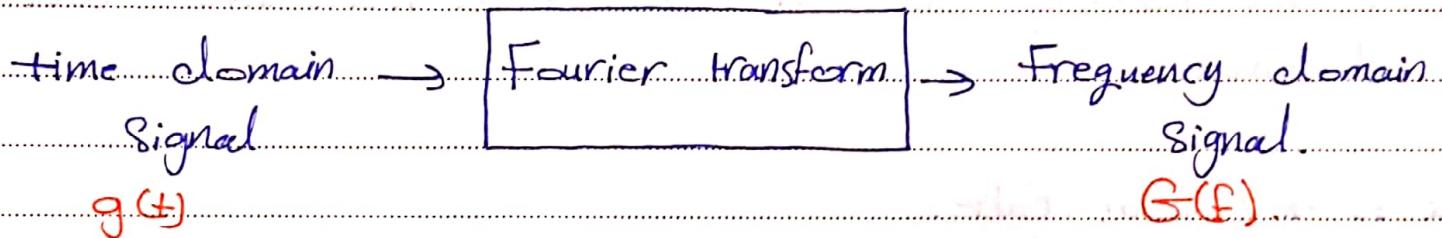
* Radio Communications (AM and FM)

- * mobile Communication.
- * Satellite Communication.
- * Wireless network (WiFi, WiMAX).

antennas used for transmission and reception. ← Wireless →

$$X(t) = A \cos \omega t$$

→ * in Communications Frequency domain is always used.



→ * let we have a time domain signal $g(t)$, and I need to convert this time domain into Frequency domain signal, so I need to use Fourier transform :-

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

→ * let I have a frequency domain signal and I need to change this signal into time domain then I need to use inverse Fourier transform :-

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

Fourier transform

time domain
description
 $g(t)$

Frequency domain
description
 $G(f)$

inverse Fourier transform

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

$$\therefore G(f) = |G(f)| e^{j\theta(f)}$$

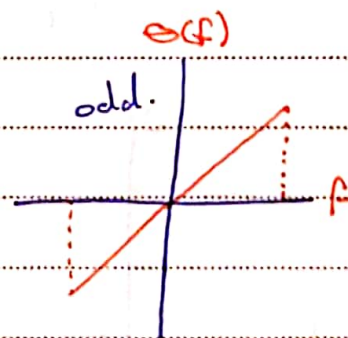
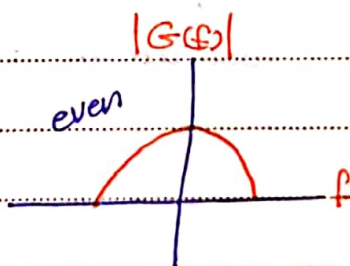
amplitude phase

* amplitude :- amplitude spectrum of signal is an even function of frequency.

* Phase :- the phase spectrum of signal is an odd function of the frequency.

القيمة على المحور y axis تكون زوجية even *

الزاوية على المحور x axis تكون فردية odd *

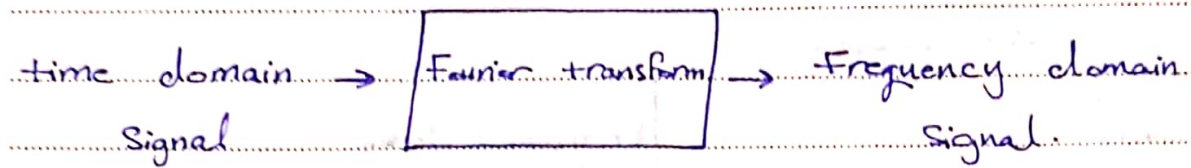


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*

chapter 2 :- Fourier transform :-



* Fourier transform is used to change the signal's representation from time domain into frequency domain.

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

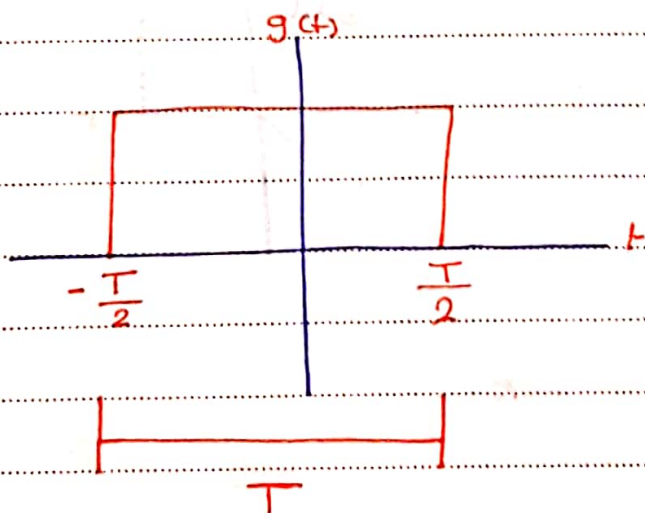
} Fourier transform pair.

* Example :-

Find the Fourier transform of the signal $g(t) = A \text{rect}\left(\frac{t}{T}\right)$.

Solution :-

$g(t) = A \text{rect}\left(\frac{t}{T}\right) \rightarrow$ amplitude = A
duration = T

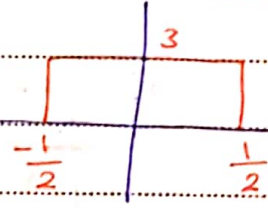




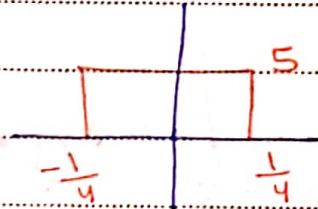
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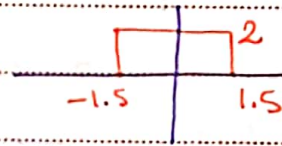
* $g(t) = 3 \text{ rect}(t)$ →



* $g(t) = 5 \text{ rect}(2t)$ →



* $g(t) = 2 \text{ rect}\left(\frac{t}{3}\right)$ →



∴ $G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$

$= \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j2\pi ft} dt$

$= A \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi ft} dt$

$= A \cdot \frac{e^{-j2\pi ft}}{-j2\pi f} \Big|_{t=-\frac{T}{2}}^{\frac{T}{2}}$

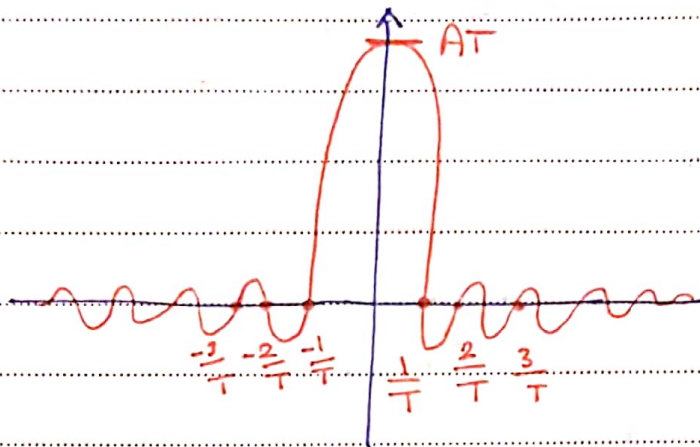
$= \frac{A}{-j2\pi f} (e^{-j\pi fT} - e^{j\pi fT}) \rightarrow \sin \pi fT = \frac{1}{2j} (e^{j\pi fT} - e^{-j\pi fT})$

$= \frac{A}{\pi f} \cdot \frac{1}{j2} (e^{j\pi fT} - e^{-j\pi fT})$

$= \frac{A}{\pi f} \sin(\pi fT) = A \frac{\sin(\pi fT)}{\pi f} \rightarrow \text{Sinc } x = \frac{\sin x}{x}$

$= AT \text{ sinc}(\pi fT)$

$$\left\{ F A \operatorname{rect}\left(\frac{t}{T}\right) \right\} \quad \text{AT} \operatorname{sinc}\left(\frac{f}{T}\right).$$

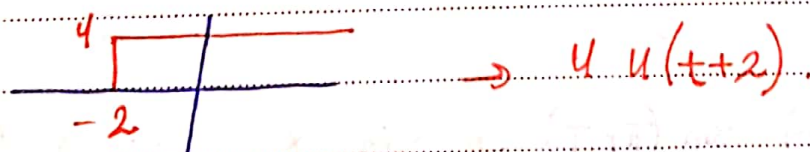
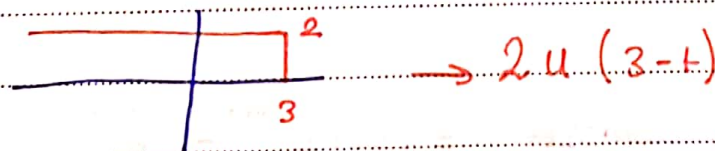
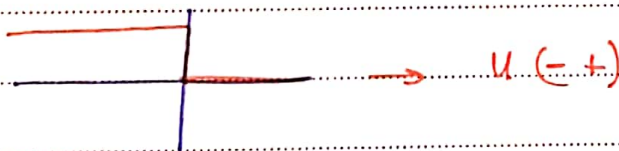
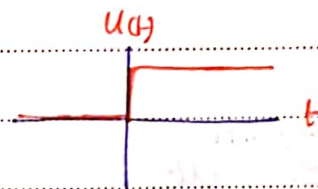


إذا طلب المقدار
تكون الإشارة الحقيقية بال موجب .

* Example : - Find the fourier transform of the signal ?

$$g(t) = e^{-at} u(t) \quad \rightarrow \quad u(t) = \text{unit step function.}$$

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



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$$\therefore g(t) = e^{-at} u(t)$$

$$= \begin{cases} e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\{e^{-\infty} = \text{Zero}\}$$

$$\therefore G(f) = \int_0^{\infty} e^{-at} e^{-j2\pi ft} dt$$

$$= \int_0^{\infty} e^{-(a+j2\pi f)t} dt$$

$$= \left. \frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right|_0^{\infty}$$

$$= \frac{1}{-(a+j2\pi f)} \left(\cancel{e^{-\infty}} - \cancel{e^0} \right)$$

Zero1

$$= \frac{-1}{-(a+j2\pi f)} = \frac{1}{a+j2\pi f}$$

Q. if

$$|G(f)| = ?$$

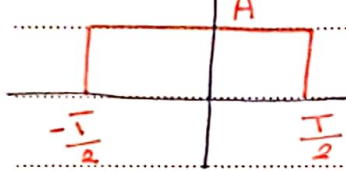
$$|G(f)|^2 = G(f) \cdot G^*(f)$$
$$= \frac{1}{a+j2\pi f} \cdot \frac{1}{a-j2\pi f}$$

$$\Rightarrow \frac{1}{a^2 + (2\pi f)^2}$$

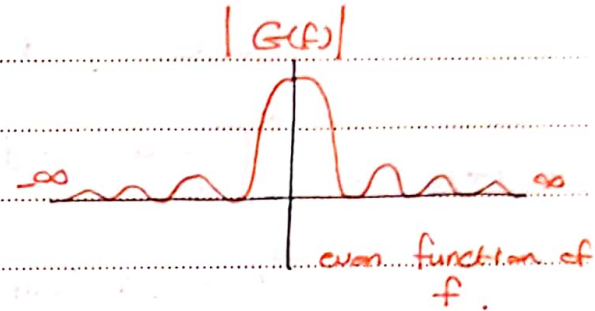
$$|G(f)| = \frac{1}{\sqrt{a^2 + (2\pi f)^2}}$$

$$* \quad G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

1) $g(t) = A \text{ rect}\left(\frac{t}{T}\right)$

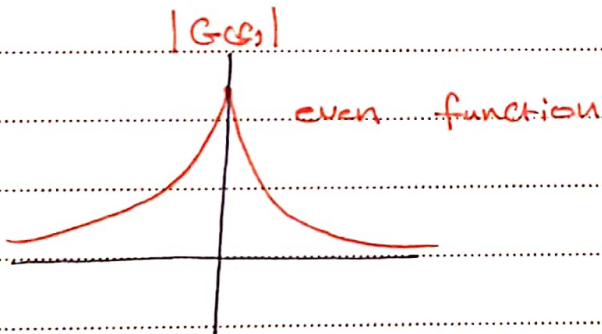


Fourier transform



2) $g(t) = e^{-at} u(t)$

$$e^{-at} u(t) \xrightarrow{\text{Fourier transform}} |G(f)| = \frac{1}{\sqrt{a^2 + (2\pi f)^2}}$$



properties of Fourier transform :-

1) Linearity (superposition) :-

$$\text{let } F\{g_1(t)\} = G_1(f)$$

$$\text{and } F\{g_2(t)\} = G_2(f)$$

$$\therefore F\{a g_1(t) + b g_2(t)\} = a G_1(f) + b G_2(f)$$

a, b are constant.

* Example :-

Compute the fourier transform of $g(t) = e^{-a|t|}$

$$|t| = \begin{cases} t & t \geq 0 \\ -t & t < 0 \end{cases}$$

$$g(t) = e^{-at} u(t) + e^{at} u(-t)$$

او جيبك في اليمين واليسار

$$F \{ e^{at} u(t) \} = \int_{-\infty}^{\infty} e^{at} e^{-j2\pi ft} dt = \frac{1}{a - j2\pi f}$$

$$F \{ e^{at} u(-t) \} = \frac{1}{a + j2\pi f}$$

$$= \frac{2a}{a^2 + (2\pi f)^2}$$

$$F \{ e^{-a|t|} \} = F \{ e^{-at} u(t) + e^{at} u(-t) \}$$

$$= \frac{2a}{a^2 + (2\pi f)^2}$$

(2) Dilation :-

$$\text{let } F \{ g(t) \} = G(f)$$

$$\text{what } F \{ g(at) \} = ?$$

$$F \{ g(at) \} = \int_{-\infty}^{\infty} g(at) e^{-j2\pi ft} dt$$



Let $u = at$, $du = a dt \rightarrow \therefore dt = \frac{du}{a}$

$$\begin{aligned} \therefore F\{g(u)\} &= \int_{-\infty}^{\infty} g(u) \cdot e^{-j2\pi f \frac{u}{a}} \frac{du}{a} \\ &= \frac{1}{a} \int_{-\infty}^{\infty} g(u) e^{-j2\pi \frac{F}{a} u} du \\ &= \frac{1}{|a|} G\left(\frac{f}{a}\right) \end{aligned}$$

$$\therefore F\{g(at)\} = \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

* Example :-

Find Fourier transform $\rightarrow \underbrace{F\{e^{-2at} u(t)\}}_{g(2t)}$

Solution :-

$$F\{e^{-at} u(t)\} = \frac{1}{a + j2\pi f} \rightarrow g(f)$$

$$\therefore F\{g(2t)\} = \frac{1}{2} G\left(\frac{f}{2}\right) = \frac{1}{2} \left(\frac{1}{a + j2\pi \frac{f}{2}} \right)$$

Problem (1) :- Find :-

1) $F\{e^{-\frac{1}{2}at} u(t)\}$

2) $F\{e^{\frac{1}{5}at} u(t)\}$

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* Example :-

Find $F \left\{ 3 \text{ rect } \frac{t}{2T} \right\}$.

Solution :-

$$F \left\{ 3 \text{ rect } \left(\frac{t}{T} \right) \right\} = 3T \text{ sinc } (\pi f T)$$

$$\therefore F \left\{ 3 \text{ rect } \left(\frac{t}{2T} \right) \right\} = g \left(\frac{1}{2} t \right)$$

$$\therefore F \left\{ g \left(\frac{1}{2} t \right) \right\} = \frac{1}{1/2} G \left(\frac{f}{1/2} \right) = 2 G(2f)$$

$$\therefore F \left\{ 3 \text{ rect } \left(\frac{t}{2T} \right) \right\} = 6T \text{ sinc } (2\pi f T)$$

* * *
Problems :- (2)

$$1) F \left\{ 4 \text{ rect } \left(\frac{t}{5T} \right) \right\}$$

$$2) F \left\{ 2 \text{ rect } \left(\frac{3t}{T} \right) \right\}$$

$$3) F \left\{ \text{rect } \left(\frac{2t}{3T} \right) \right\}$$

Problem (1) :-

1) $F \{ e^{-\frac{1}{2}at} u(t) \}$

$$g(t) = e^{-at} u(t) \rightarrow F(g(t)) = \frac{1}{a + j2\pi f}$$

$$\therefore F \{ e^{-\frac{1}{2}at} u(t) \} = F \{ g(\frac{-1}{2}t) \}$$

$$= \frac{1}{|\frac{1}{2}|} G\left(\frac{f}{\frac{1}{2}}\right) = 2 G(2f) = 2 \left(\frac{-1}{a - j4\pi f} \right)$$

2) $F \{ e^{\frac{1}{5}at} u(t) \}$

$$g(t) = e^{at} u(t) \rightarrow F(g(t)) = \frac{1}{a - j2\pi f}$$

$$\therefore F \{ e^{\frac{1}{5}at} u(t) \} = F \{ g(\frac{1}{5}t) \}$$

$$= \frac{1}{|\frac{1}{5}|} G\left(\frac{f}{\frac{1}{5}}\right) = 5 G(5f) = 5 \left(\frac{1}{a + j2\pi(5f)} \right)$$

Problem (2) :-

1) $F \left(4 \text{ rect } \frac{t}{5T} \right)$

$$F \left(4 \text{ rect } \frac{t}{5T} \right) = 4T \text{ sinc}(\pi f T) \rightarrow g(t)$$

$$F \left(4 \text{ rect } \frac{t}{5T} \right) = g\left(\frac{1}{5}t\right) = \frac{1}{|1/5|} G\left(\frac{f}{1/5}\right)$$

$$= 5 G(5f) = 20T \text{ sinc}(5\pi f T)$$

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2) $F\left(2 \text{ rect } \frac{3t}{T}\right)$

$$F\left(2 \text{ rect } \frac{t}{T}\right) = 2T \text{ sinc}(\pi f T) \rightarrow g(t)$$

$$F\left(2 \text{ rect } 3 \frac{t}{T}\right) = g\left(3t\right) = \frac{1}{|3|} G\left(\frac{F}{3}\right)$$

$$= \frac{2}{3} T \text{ sinc}\left(\frac{\pi F T}{3}\right)$$

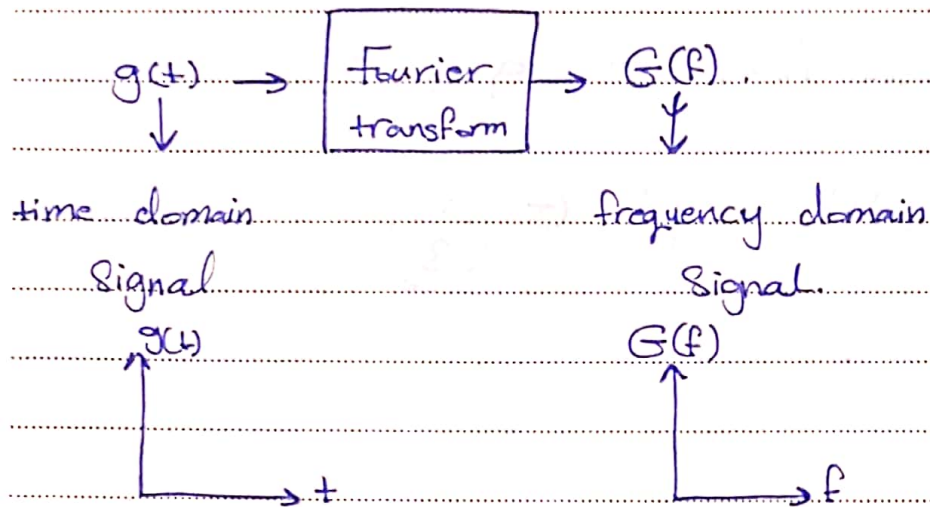
3) $F\left(\text{rect } \frac{2t}{3T}\right)$

$$F\left(\text{rect } \frac{t}{T}\right) = T \text{ sinc}(\pi f T) \rightarrow g(t)$$

$$\therefore F\left(\text{rect } \frac{2}{3} \frac{t}{T}\right) = g\left(\frac{2}{3}t\right) = \frac{1}{|2/3|} G\left(\frac{F}{2/3}\right)$$

$$= \frac{3}{2} T \text{ rect}\left(T \frac{3F}{2} T\right)$$

* Fourier transform :-



properties of Fourier transform :-

- 1) linearity (superposition)
- 2) Dilation.
- 3) Time shifting :-

Shifting a signal in time domain $\rightarrow$ result in frequency domain

$$g(t) \rightarrow G(f)$$

$$g(t-t_0) \rightarrow ?$$

$$G(f) = \int_{-\infty}^{\infty} g(t-t_0) e^{-j2\pi ft} dt$$

$$\text{let } u = t - t_0 \rightarrow du = dt \rightarrow t = u + t_0$$

$$\therefore G(f) = \int_{-\infty}^{\infty} g(u) e^{-j2\pi f(u+t_0)} du$$

$$= \int_{-\infty}^{\infty} g(u) e^{-j2\pi fu} \underbrace{e^{-j2\pi ft_0}}_{\text{Constant}} du$$

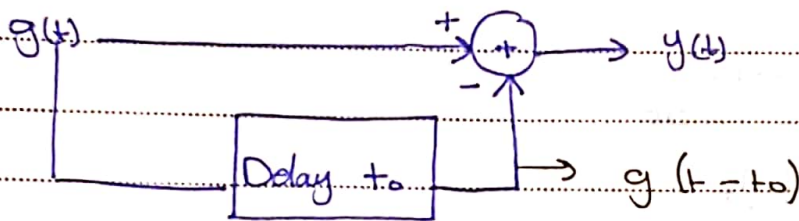
$$= \int_{-\infty}^{\infty} e^{-j2\pi ft_0} \int_{-\infty}^{\infty} g(u) e^{-j2\pi fu} du$$

$$= e^{-j2\pi ft_0} \cdot G(f)$$

$$\therefore g(t-t_0) \Rightarrow e^{-j2\pi ft_0} G(f)$$

Example :-

for the block diagram shown below find the transfer function?



$$T(f) = \frac{Y(f)}{G(f)}$$

$$\mathcal{F}\{g(t)\} = G(f)$$

$$\mathcal{F}\{y(t)\} = Y(f)$$

$$\mathcal{F}\{g(t-t_0)\} = e^{-j2\pi ft_0} G(f)$$

$$y(t) = g(t) - g(t-t_0)$$

$$Y(f) = G(f) - G(f) e^{-j2\pi ft_0}$$

$$Y(f) = G(f) [1 - e^{-j2\pi ft_0}]$$

$$\therefore T(f) = \frac{Y(f)}{G(f)} = 1 - e^{-j2\pi ft_0}$$

4) Frequency Shifting :-

$$g(t) \rightarrow G(f)$$

$$e^{+j2\pi f_c t} g(t) \rightarrow ??$$

$\downarrow$ $\downarrow$
 $X(t)$ $X(f)$

$$\therefore X(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{j2\pi f_c t} \cdot e^{-j2\pi f t} dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{-j2\pi (f - f_c) t} dt$$

$$= G(f - f_c)$$

$$\therefore e^{j2\pi f_c t} g(t) \Rightarrow G(f - f_c)$$

* Example :-

Find the Fourier transform of $X(t) = 2 \cos 2\pi f_c t g(t)$

$$* 2 \cos 2\pi f_c t = e^{j2\pi f_c t} + e^{-j2\pi f_c t}$$

$$\therefore X(t) = e^{j2\pi f_c t} g(t) + e^{-j2\pi f_c t} g(t)$$

$$\therefore F(X(t)) = F\{e^{j2\pi f_c t} g(t)\} + F\{e^{-j2\pi f_c t} g(t)\}$$

$$= G(f - f_c) + G(f + f_c)$$

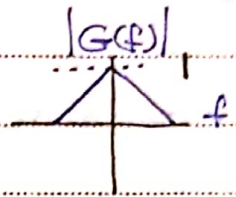
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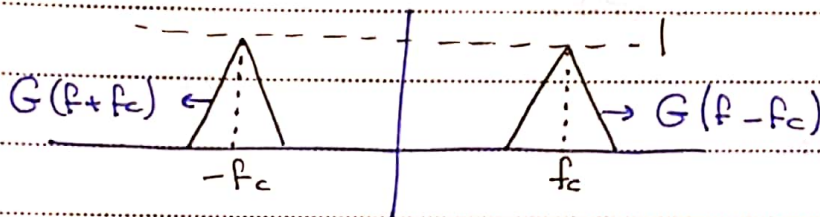
Example :-

let the signal $g(t)$ has a fourier transform as in the figure
how the fourier transform of



$x(t) = 2 \cos 2\pi f_c t g(t)$ looks like
(draw the fourier transform of $x(t)$).

∴ frequency shift $\Rightarrow$ $G(f \pm f_c)$



5) Differentiation in time domain :-

$$g(t) \rightarrow G(f)$$

$$\frac{d}{dt} g(t) \rightarrow j2\pi f G(f)$$

$$\frac{d^n}{dt^n} g(t) \rightarrow (j2\pi f)^n G(f)$$

* Example :- Find fourier transform of $g(t) = t$

$$G(f) = \int_{-\infty}^{\infty} t e^{-j2\pi f t} dt$$

$$g(t) = t \rightarrow \bar{g}(t) = 1 \rightarrow F\{1\} =$$

$$\therefore F(g(t)) = F\{1\} (j2\pi f)$$

$$g(t) = t^2$$

$$F\{g(t)\} = (j2\pi f)^2 F\{t^2\}$$

* Properties of Fourier Transform :-

1) Time shifting :-

$$F\{g(t - t_0)\} \rightarrow e^{-j2\pi f t_0} G(f)$$

2) Frequency shifting :-

$$g(t) e^{j2\pi f_0 t} \rightarrow G(f - f_0)$$

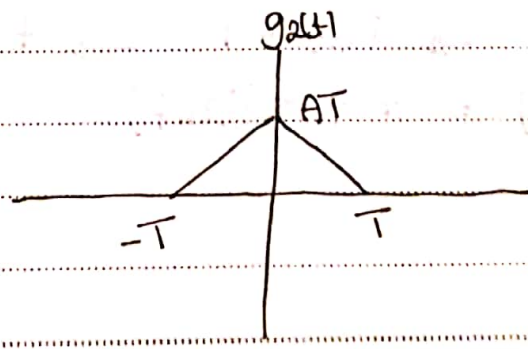
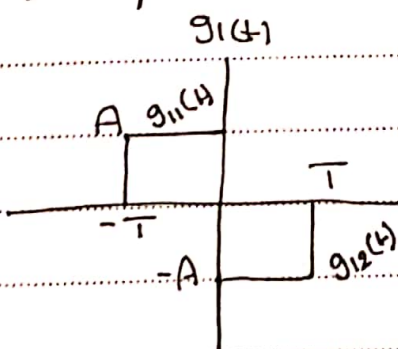
3) Differentiation in time domain :-

$$\dot{g}(t) \rightarrow (j2\pi f) G(f)$$

4) integration in time domain :-

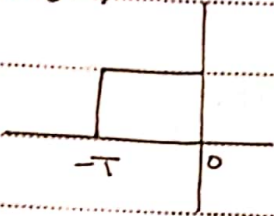
$$\int_{-\infty}^t g(\tau) d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f)$$

* Example :-



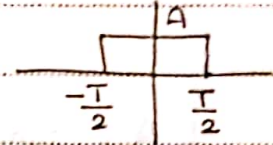
في الحالة g_1 و g_2 ← g_1, g_2 ليكن g_2 ليكن

$(-T, 0)$

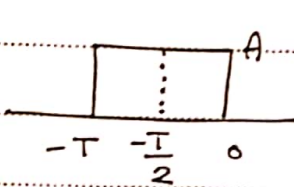


↓

in general :



$$F\left(A \text{ rect } \frac{t}{T}\right) = AT \text{ sinc}(TfT)$$



→ $A \text{ rect } \left(\frac{t + T/2}{T}\right)$ shift in time domain.

$$F\left\{A \text{ rect } \left(\frac{t + T/2}{T}\right)\right\}$$

$$= AT \text{ sinc}(\pi f T) e^{j2\pi f \frac{T}{2}}$$

$$= AT \text{ sinc}(\pi f T) e^{j\pi f T}$$

$g_1(t) (0, T) :$

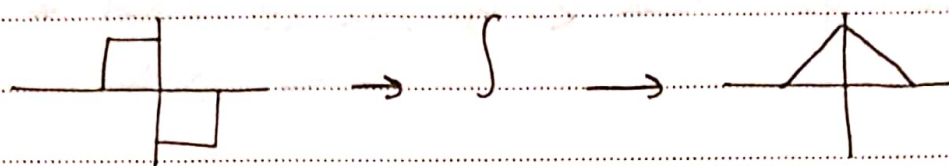
$$g(t) = -A \text{ rect } \left(\frac{t - T/2}{T}\right)$$

$$F(g_{12}(t)) = -AT \text{ sinc}(\pi f T) e^{-j\pi f T}$$

$$F(g_2(t)) = \frac{1}{j2\pi f} F(g_1(t)) = \frac{1}{j2\pi f} F(g_{11}(t) + g_{12}(t))$$

$$= \frac{1}{j2\pi f} (F(g_{11}(t)) + F(g_{12}(t)))$$

$$= \frac{1}{j2\pi f} (AT \text{ sinc}(\pi f T) e^{j\pi f T} - AT \text{ sinc}(\pi f T) e^{-j\pi f T})$$



$$\therefore = \frac{1}{j2\pi f} \left[AT \operatorname{sinc}(\pi f T) e^{j\pi f T} - AT \operatorname{sinc}(\pi f T) e^{-j\pi f T} \right]$$

$$= \frac{AT \operatorname{sinc}(\pi f T) \left[e^{j\pi f T} - e^{-j\pi f T} \right]}{j2\pi f}$$

Note :- $\sin \pi f T = \frac{1}{2j} \left[e^{j\pi f T} - e^{-j\pi f T} \right]$

$$= AT \operatorname{sinc}(\pi f T) \cdot \frac{2j \sin(\pi f T)}{j2\pi f} \rightarrow \frac{T}{T}$$

$$= AT \operatorname{sinc}(\pi f T) \cdot T \cdot \frac{\sin(\pi f T)}{\pi f T}$$

$$= AT^2 \operatorname{sinc}^2(\pi f T)$$



$$F\{g_2(t)\} = \frac{1}{j2\pi f} F\{g_1(t)\}$$

* Properties of Fourier transform :-

Modulation property :-

$$\text{let } g_1(t) \rightarrow G_1(f)$$

$$g_2(t) \rightarrow G_2(f)$$

Then what is the Fourier transform of $F\{g_1(t) g_2(t)\}$.

$$F\{g_1(t) g_2(t)\} = G_1(f) \underset{\text{Convolution}}{*} G_2(f)$$

Convolution Theory :-

$$F\{g_1(t) \underset{\text{Convolution}}{*} g_2(t)\} = G_1(f) G_2(f)$$

Duality :-

$$\text{let } g_1(t) \rightleftharpoons G_1(f)$$

$$G(f) \rightleftharpoons g(-f)$$

$$F\left\{A \text{rect}\left(\frac{t}{T}\right)\right\} = AT \text{sinc}(TfT)$$

Find $F\{A \text{sinc}(2\omega t)\}$ $\rightarrow$ 2nd part of the duality property we use duality

$$\begin{aligned} \text{Ans } F\{A \text{sinc}(2\omega t)\} &= A \frac{1}{2\omega} \text{rect}\left(f \cdot \frac{1}{2\omega}\right) \\ &= \frac{A}{2\omega} \text{rect}\left(\frac{f}{2\omega}\right) \end{aligned}$$

or : $A \operatorname{sinc}(2\omega t) = A \frac{\sin 2\omega t}{2\omega t} \rightarrow$ جولف

Rayleigh energy theorem :-

let we have the fourier transform pair

$$g(t) \rightleftharpoons G(f)$$

→ energy contained in the signal g(t)

Then $\int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$

$\therefore \underset{\text{energy}}{E} = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$

* Example :-

Compute the energy of the signal $g(t) = A \operatorname{sinc}(2\omega t)$

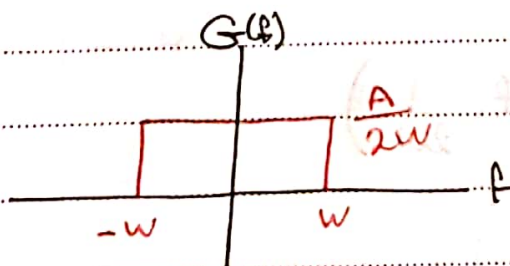
$$E = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$



$$\operatorname{sinc}(2\omega t) = \frac{\sin 2\omega t}{2\omega t}$$

جولف

or : $E \{ A \sin(2\omega t) \} = \frac{A}{2\omega} \operatorname{rect}\left(\frac{f}{2\omega}\right)$



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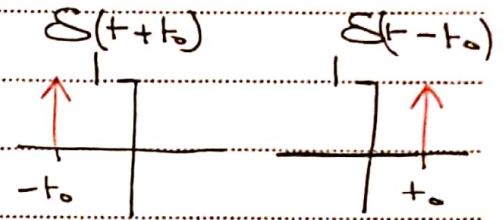
$$= \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$= \int_{-w}^w \left(\frac{A}{2w}\right)^2 df = \left(\frac{A}{2w}\right)^2 \int_{-w}^w df = \left(\frac{A}{2w}\right)^2 \cdot f \Big|_{-w}^w$$

$$= \left(\frac{A}{2w}\right)^2 \cdot 2w = \frac{A^2}{2w}$$

Section (2.4) :- delta function :-

$$\delta(t - t_0) = \begin{cases} 1, & t = t_0 \\ 0, & \text{otherwise} \end{cases}$$



$$\int_{-\infty}^{\infty} g(t) \delta(t - t_0) dt = g(t_0)$$

Convolution

$$* g(t) * \delta(t) = g(t)$$

$$\rightarrow F\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = e^{-j2\pi ft_0} \Big|_{t_0=0} = 1$$

$$\therefore F(\delta(t)) = 1$$

$$F\left\{ \begin{array}{c} \delta(t) \\ \uparrow \\ t \end{array} \right\} = \begin{array}{c} 1 \\ \hline f \end{array} \rightarrow \text{it has a constant value for the whole frequency band.}$$

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$
$$= \int_{-\infty}^{\infty} \delta(f) e^{j2\pi ft} df = e^{j2\pi ft} \Big|_{f=0} = 1$$