Subject : 26 / 2 / 2019.

Fourier transform (Delta function) :-

$$\mathcal{F}\{1\} = \delta(f).$$

$$\delta(f) = \int_{-\infty}^{\infty} e^{-j2\pi ft}\, dt.$$

Ex.:- $\mathcal{F}\{e^{j2\pi f_c t}\}$.

$$= \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} e^{j2\pi f_c t} \cdot e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-j2\pi (f-f_c)t}\, dt$$

$$= \delta(f-f_c).$$

*** Ex.:- Find fourier transform of $g(t) = \cos 2\pi f_c t$.**

$$g(t) = \frac{1}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right).$$

$$\therefore \mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}\mathcal{F}\{e^{j2\pi f_c t}\} + \frac{1}{2}\mathcal{F}\{e^{-j2\pi f_c t}\}.$$

$$= \frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)$$

$$= \frac{1}{2}\{\delta(f-f_c) + \delta(f+f_c)\}.$$

Subject : 26 / 2 / 2017

*Ex : Compute $\mathcal{F}\{2\sin(2\pi f_c t)\}$

$$\therefore \mathcal{F}\{2\sin(2\pi f_c t)\} = \frac{1}{j} e^{j2\pi f_c t} - \frac{1}{j} e^{-j2\pi f_c t},$$

$$= \frac{1}{j}\,\delta(f-f_c) - \frac{1}{j}\,\delta(f+f_c).$$

problem :

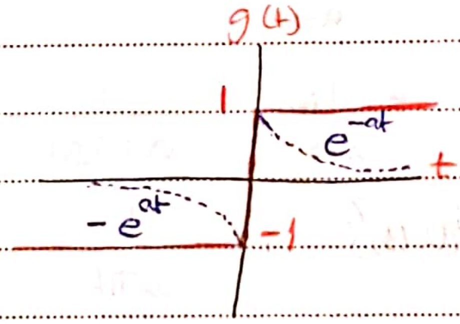
Compute fourier transform of :

1) $g(t) = 2\sin^2(2\pi f t)$.
2) $g(t) = 3\sin^2(6\pi f t)$.
3) $g(t) = \cos^2(2\pi f t)$.

*Example : $\mathcal{F}\{\operatorname{sgn}(t)\}$

$$g(t) = \operatorname{sgn}(t)$$

$$= \begin{cases} 1 , & t \geqslant 0 \\ -1 , & t < 0 \end{cases}$$



$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j2\pi f}.$$

$$\mathcal{F}\{-e^{at}u(-t)\} = \frac{-1}{a-j2\pi f}$$

$$\therefore \frac{1}{a+j2\pi f} - \frac{1}{a-j2\pi f} = \frac{a - j2\pi f - a - j2\pi f}{(a+j2\pi f)(a-j2\pi f)}$$

$$= \frac{-j4\pi f}{a^2 + (2\pi f)^2}$$

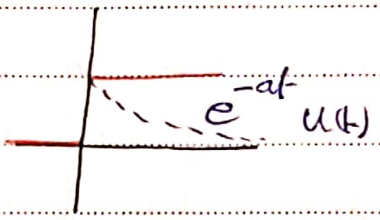
Subject : 26 / 2 / 2019

$$\therefore F\{\mathrm{Sgn}(t)\} = \lim_{a\to 0}\left[\frac{-j4\pi f}{a^2+(2\pi f)^2}\right]$$

$$= -j2 \cdot \frac{2\pi f}{(2\pi f)^2} = \frac{-j2}{2\pi f} = \frac{-j}{\pi f} = \frac{1}{j\pi f}$$

$* \; g(t) = u(t)$

$$= \begin{cases} 1 & t \geq 0 \\ 0 & o.w \end{cases}$$



$$F\{u(t)\} = \underset{a\to 0}{F}\{e^{-at}u(t)\}$$

$$= \lim_{a\to 0} \frac{1}{a+j2\pi f}$$

$$* \; \therefore F\{u(t)\} = \frac{1}{j2\pi f} \; *$$

Fourier → Transform :- For aperiodic Signal.
Fourier → Series :- For periodic Signal.

Subject : Filters :   27 / 2 / 2019 .

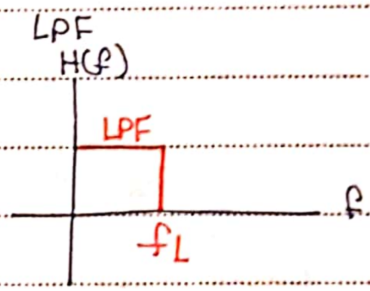
1) low pass filter (LPF).

2) High pass filter (HPF).

3) Band pass filter (BPF).

4) Band stop filter (BSF).

LPF :



This filter lets all signals that have frequences in the range $0 \leq f \leq f_L$ to pass through this filter

* Example :

let we have a LPF with $F_L = 20$ kHz . check which of the following signals can pass through the filter ?

1) $H(t) = 3 \cos 30000 \pi t$.
2) $y(t) = \sin 50000 \pi t$.
3) $z(t) = \sin^2 25000 \pi t$.
4) $w(t) = 2 \cos 30000 \pi t \sin 30000 \pi t$.

Solution :

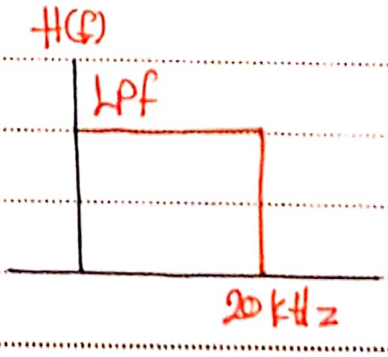
1) $H(t) = 3 \cos 30000 \pi t$.

$H(t) = \cos 2\pi f t$

$= \cos 2(15 \text{ kHz}) \pi t$

$0 < 15 \text{ kHz} < f_L = 20 \text{ kHz}$.

This filter can pass from the filter



Subject : 27 / 2 / 2019.

2) $y(t) = \sin 50000\pi t$

$= \sin (25\ kHz)\ 2\pi t$

$25\ kHz > f_L$

$\therefore$ The LPF will filter out the signal.

3) $z(t) = \sin^2 25000\pi t$

$= \frac{1}{2} + \frac{1}{2} \sin 2\pi (2f) t$

$= \frac{1}{2} + \frac{1}{2} \sin 2\pi (50\ kHz) t$

$= \underbrace{\frac{1}{2}} + \underbrace{\frac{1}{2} \sin 2\pi (50\ kHz) t}$

dc value ↲ ↳ AC with $f = 50\ kHz$

$f = 0\ Hz$

4) $w(t) = 2 \cos 30000\pi t \ \sin 3000\pi t$

$= \cancel{\ } \sin 60000\pi t$

$= \sin 2\pi t\ (30\ kHz)$

$30\ kHz > f_L$

↳ This signal ~~is~~ is filtered out.

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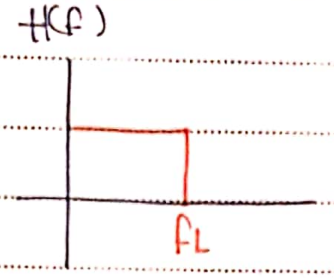
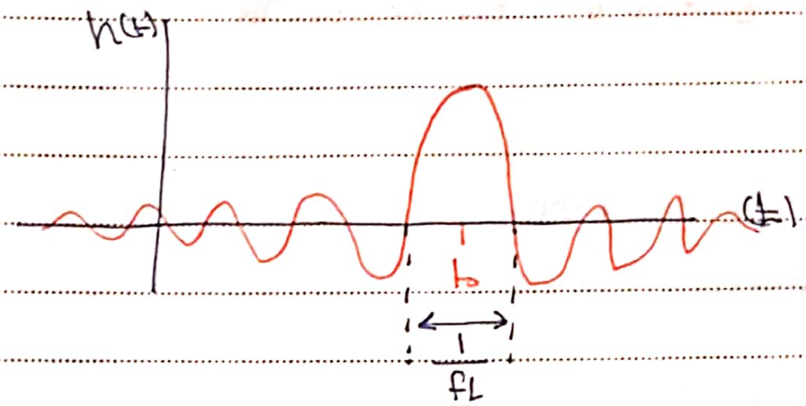
Subject : .... 27 / 2 / 2019



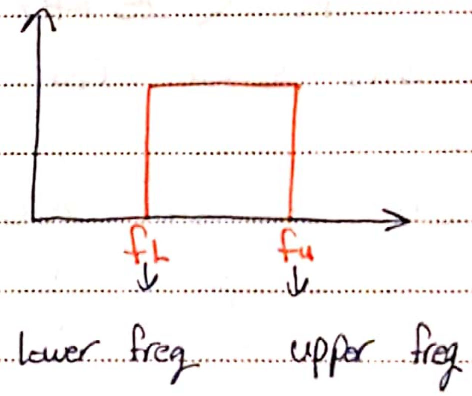
$$H(f) = \begin{cases} e^{-j2\pi f t_0} & -f_L \leq f \leq f_L \\ 0 & o.w \end{cases}$$

$$h(t) = 2f_L \, \text{sinc}\left(2f_L (t - t_0)\right)$$



BPF :-

Band pass filter will let all signals that have frequinces between f_L and f_u to passes.



* Example :-

let we have BPF with frequency $F_L = 20\ kHz$, $F_u = 40\ kHz$, check which of the following signals can pass through the filter ?

Subject : 27/2/2019

1) $H(t) = 3 \cos 30000 \pi t$.

2) $y(t) = \sin 50000 \pi t$.

3) $Z(t) = \sin^2 25000 \pi t$.

4) $W(t) = 2 \cos 30000 \pi t \sin 30000 \pi t$.

Solution :-

1) $H(t) = 3 \cos (15\ kHz) 2\pi t$

$\therefore$ it will be filtered out.

2) $y(t) = \sin (25\ kHz) 2\pi t$

$f_L < 25\ kHz < f_u$

$\therefore$ This signal can passes from this filter.

3) $Z(t) = \sin^2 25000 \pi t$.

$= \underbrace{\frac{1}{2}}_{} + \frac{1}{2} \underbrace{\sin 50000 \pi t}_{\text{AC value}}$.

$f = 0\ Hz$ DC value

$f = 25000\ Hz$.

4) $W(t) = 2 \cos 30000 \pi t \sin 30000 \pi t$.

$= \sin 60000 \pi t$.

$\therefore$ Can pass through the filter.