(4) Given that 1 has been received, what is the probability that 1 was transmitted?

$$P[T_1/R_1] = \frac{P[T_1 \cap R_1]}{P[R_1]} = \frac{P[R_1/T_1]\,P[T_1]}{P[R_1]}$$

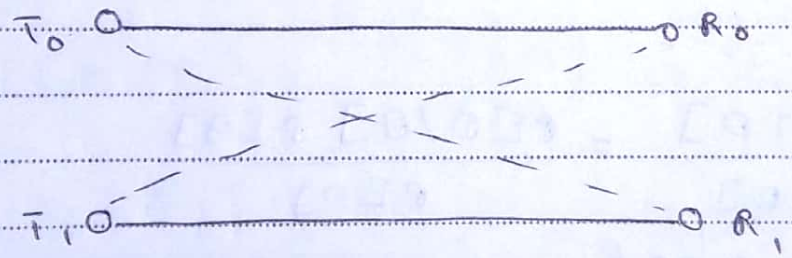
$$= \frac{(0.98)(0.4)}{0.398} = \frac{0.392}{0.398}$$

(5) what is the probability of Error?

$$P[E] = P[T_0 \cap R_1] + P[T_1 \cap R_0]$$
$$= P[R_1/T_0]\,P[T_0] + P[R_0/T_1]\,P[T_1]$$
$$= (0.01)(0.6) + (0.02)(0.4) = 0.006 + 0.008 = 0.014$$

Total Probability and Bay's Rule

Binary communication channel



EX/ A company producing electric relays has manufacturing plans producing 45/30 and 25 percent, respectively, of its products. Suppose that a relay manufactured by these plants is defective with probability 0.015, 0.02 and 0.07

1) Suppose that a relay is selected @ random,

what is the Probability that it is defective?

غير قابلة للاستقلال

$A \rightarrow P[A] = 0.45$

$B \rightarrow P[B] = 0.3$

$C \rightarrow P[C] = 0.25$

$P[D/A] = 0.015 \qquad P[D/B] = 0.02$

$P[D/C] = 0.07$

Total probability to find the probability of defective

$P[D] = P[D \cap A] + P[D \cap B] + P[D \cap C]$

$= P[D/A]\,P[A] + P[D/B]\,P[B] + P[D/C]\,P[C]$

$= (0.015)(0.45) + (0.3)(0.02) + (0.07)(0.25) = 0.03025$

Bay's Rule :-

* Given that the selected part is defective, what is the probability that this part is from line B ?

$$P[B/D] = \frac{P[B \cap D]}{P[D]} = \frac{P[D/B]\,P[B]}{P[D]}$$

$$= \frac{(0.02)(0.3)}{0.03025} = \frac{0.006}{0.03025}$$

EX/ At a certain university, 4% of men are over 6 feet tall, and 1% of women are over 6 feet tall. The total student population is divided in the ration 3:2 in favour of women.

Subject: .... / /

1) suppose a student is randomely selected what is the probability the student has over 6 feet tall ?

3 : 2

f : Female

$P[F] = 3/5$ $P[m] = 2/5$ m : male

$P[T/m] = 0.04$, $P[T/F] = 0.01$

$P[T] = P[T \cap M] + P[T \cap f]$

$= P[T/m]\, P[m] + P[T/F]\, P[F]$

$= \left(\frac{4}{100}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{100}\right)\left(\frac{3}{5}\right) = \frac{8}{500} + \frac{3}{500} = \frac{11}{500}$

2) Given that the selected student is tall, what is the probability that the student is male ?

$P[M/T] = \frac{P[T \cap M]}{P[T]} = \frac{P[T/M]\, P[m]}{P[T]}$

$= \frac{\left(\frac{4}{100}\right)\left(\frac{2}{5}\right)}{\frac{11}{500}} = \frac{8}{11}$.

Independent Events حوادث مستقلة

def: Two events A and B are said to be independent iff

$P[A \cap B] = P[A]\, P[B]$

Subject:........................ / /

$P[A \cap B] = 0$ disjoint Events

$P[A \cap B] = P[A] + P[B] - P[A \cup B]$

in the case no information such disjoint or independent is given.

EX/ Let the sample spce $S = \{1,2,3,4,5,6,7,8\}$ let event $A = \{1,2,3,4\}$, event $B = \{3,4,5,6\}$ Are A and B independent events?

(1)

$P[A] = \frac{|A|}{|S|} = \frac{4}{8} = \frac{1}{2}$

$P[B] = \frac{|B|}{|S|} = \frac{4}{8} = \frac{1}{2}$

$P[A]\,P[B]$

$(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

(2) $A \cap B = \{3,4\}$

$P[A \cap B] = \frac{|A \cap B|}{|S|} = \frac{1}{4}$

since $P[A]P[B] = P[A \cap B]$
A and B are independent Events

EX/ Let the sample spce $S = \{1,2,3,4,5,6,7,8\}$
Let event $A = \{1,2,3,4\}$, event $B = \{3,4,5,6\}$
$C = \{5,6,7,8\}$

1) find a relation between the events A and C?

2) find a relation between the events B and c?

1) $A \cap C = \{\} \Rightarrow P[A \cap C] = 0$

A and C are disjoint Events

2) $B \cap C = \{5, 6\} \Rightarrow P[B \cap C] = \frac{1}{4}$

$P[B] \; P[C] \quad = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$

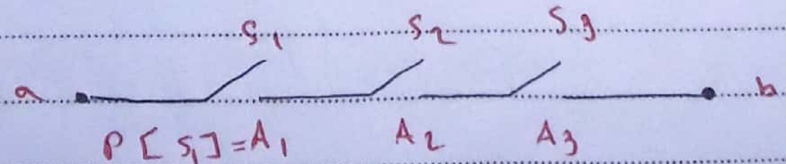
B and C are independent events

def: Let the events A and B independent
Then, the following events are independent

1) $\bar{A}, B$: independent.
2) $A, \bar{B}$: independent.
3) $\bar{A}, \bar{B}$: independent.

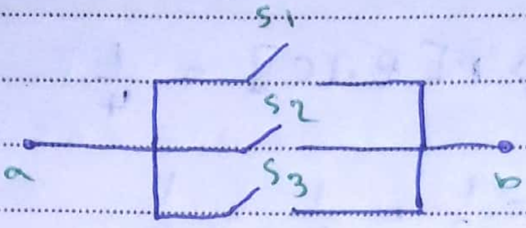
flao #

Ex/ consider the switching network in the attached figure with switches S_1, S_2 and S_3. Let A_1, A_2 and A_3 denote the events that the switches S_1, S_2 and S_3 are closed. Let A_{ab} be the event that there is a closed path between terminals a and b. Express A_{ab} in terms of A_1, A_2 and A_3?



$$P[A_{ab}] = P[S_1 \cap S_2 \cap S_3]$$
$$= P[S_1]\, P[S_2]\, P[S_3] = A_1 A_2 A_3$$

S_1, S_2 and S_3 can operate independtly



$$A_{ab} = 1 - \overline{A_{ab}}$$
$$\overline{A_{ab}} = P[\overline{S_1} \cap \overline{S_2} \cap \overline{S_3}]$$
$$P[\overline{S_1}]\, P[\overline{S_2}]\, P[\overline{S_3}]$$
$$= \overline{A_1}\, \overline{A_2}\, \overline{A_3}$$