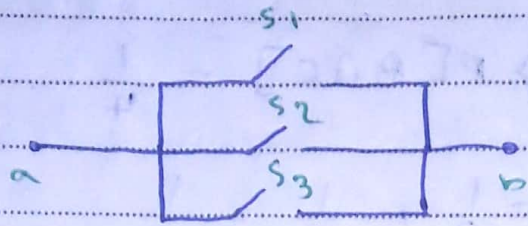


Subject: / /

$$P[A_{ab}] = P[S_1 \cap S_2 \cap S_3] \\ = P[S_1] P[S_2] P[S_3] = A_1 A_2 A_3$$

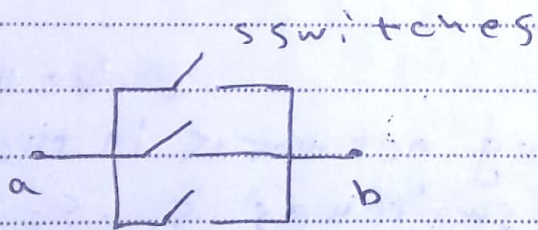
S_1, S_2 and S_3 can operate independently



$$A_{ab} = 1 - \overline{A_{ab}}$$

$$\overline{A_{ab}} = P[\overline{S_1} \cap \overline{S_2} \cap \overline{S_3}] \\ = P[\overline{S_1}] P[\overline{S_2}] P[\overline{S_3}] \\ = \overline{A_1} \overline{A_2} \overline{A_3}$$

Independent Events [system reliability]



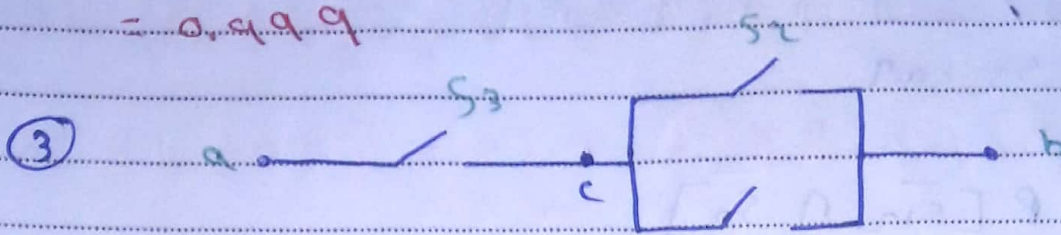
Assume that each switch can operate with probability $= a, q$ what is the probability that the path from point a to point b can function?

$$P_{ab} = 1 - \overline{P_{ab}} \\ = 1 - P[\overline{S_1} \cap \overline{S_2} \cap \overline{S_3}] \\ = 1 - P[\overline{S_1}] P[\overline{S_2}] P[\overline{S_3}]$$

$$= 1 - (0.1)(0.1)(0.1)$$

$$= 1 - 0.001$$

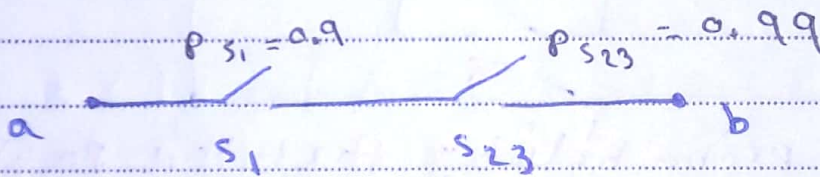
$$= 0.999$$



S_2 and S_3 are independent events.

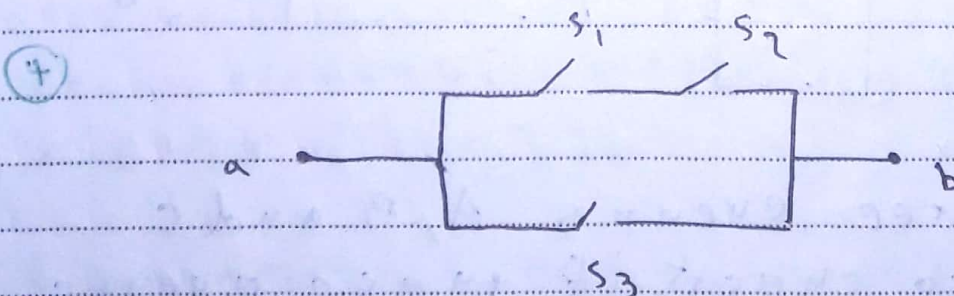
$$\begin{aligned} P_{cb} &= 1 - P[\bar{S}_2 \cap \bar{S}_3] \\ &= 1 - P[\bar{S}_2]P[\bar{S}_3] \\ &= 1 - (0.1)(0.1) \\ &= 0.99 \end{aligned}$$

→ probability that the path from point c to point b can function



$$P_{ab} = P[S_1 \cap S_{23}]$$

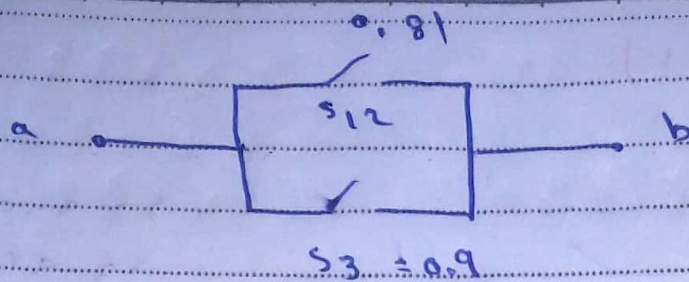
$$= P[S_1] P[S_{23}] = (0.9)(0.99) =$$



$$P[S_1 \cap S_2] = P[S_1] P[S_2]$$

$$= (0.9)(0.9) = 0.81$$

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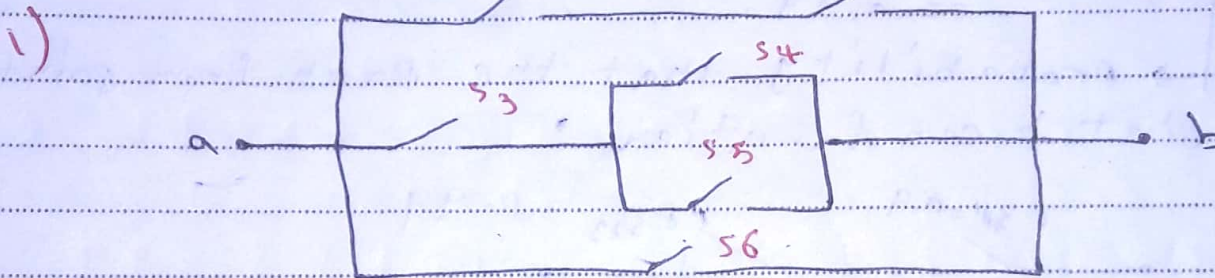


$$P_{ab} = 1 - P[\bar{s}_{12} \cap \bar{s}_3]$$

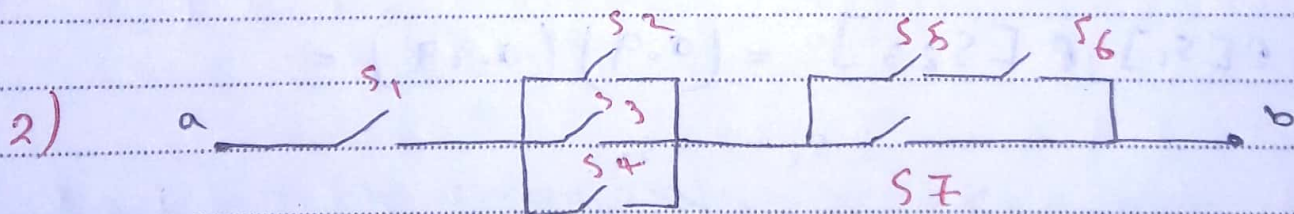
$$= 1 - P[\bar{s}_{12}] P[\bar{s}_3]$$

$$= 1 - (0.14)(0.1) = 1 - 0.014 = 0.986$$

Problems



Find the probability that the path from point a to point b can function?



We have three events A, B and C we need to check if these events are independent?

The events A, B and C are ~~ix~~ said to

Subject:

/ /

be independent of

مستقلاً عن

متغيرين

Independent of

$$1) P[A \cap B] = P[A] P[B]$$

$$2) P[A \cap C] = P[A] P[C]$$

$$3) P[B \cap C] = P[B] P[C]$$

$$4) P[A \cap B \cap C] = P[A] P[B] P[C]$$

ch 2: Random variables [r.v]

$$S = \{ \text{---}, \text{---}, \text{---}, \text{---} \}$$

$$S = \{ HH, HT, TH, TT \}$$

real numbers

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array}$$

random variable: mapping from sample space into real numbers

[each element in the sample space is given a probability]

An experiment can be modelled by using cumulative density function [CDF]

→ real values.

$f(x)$

$f_x(0), f_x(1), f_x(0.5), f_x(0.3)$

→ random variable.

S

T

A

R

S Notebook

Subject: / /

$$P[X \leq x] = F_X(x)$$

$$P[X > x] = 1 - P[X \leq x]$$

A CDF is given by

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x + 0.5 & 0 \leq x < 0.5 \\ 1 & x \geq 0.5 \end{cases}$$

$$\begin{aligned} \textcircled{1} \quad P[X \leq 0.2] &= F_X(0.2) \\ &= 0.2 + 0.5 = 0.7 \end{aligned}$$

$$\textcircled{2} \quad P[X \leq 0.8] = F_X(0.8) = 1$$

$$\begin{aligned} P[a \leq X \leq b] &= P[X \leq b] - P[X \leq a] \\ &= F_X(b) - F_X(a) \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad P[0.1 \leq X \leq 0.4] &= F_X(0.4) - F_X(0.1) \\ &= 0.9 - 0.6 = 0.3 \end{aligned}$$

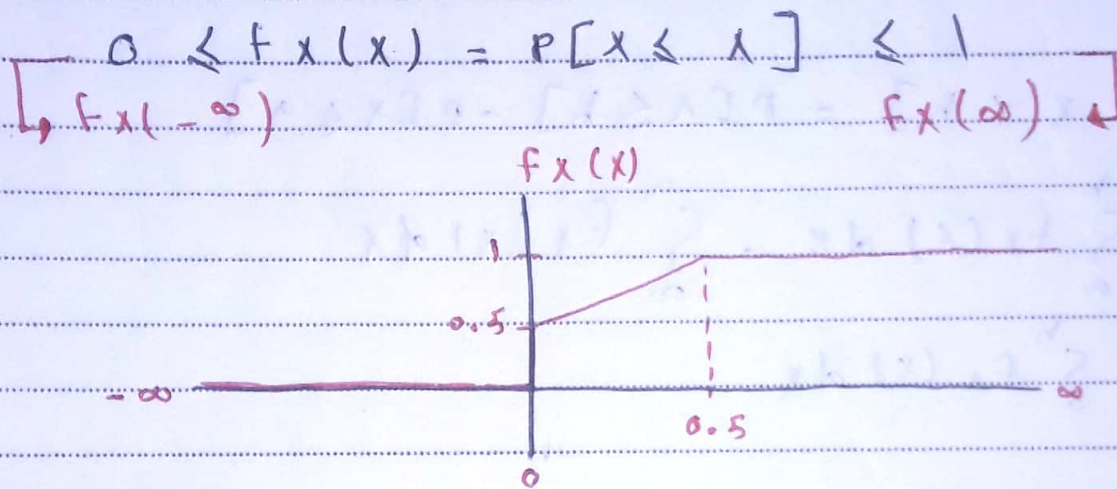
$$\begin{aligned} \textcircled{4} \quad P[0.2 \leq X \leq 0.6] &= P[0.2 \leq X \leq 0.6] = P[X \leq 0.6] - P[X \leq 0.2] \\ &= 1 - 0.7 = 0.3 \end{aligned}$$

$$F_X(x) = P[X \leq x] = P[X > x] + P[X \leq x] = 1$$

$$\textcircled{5} P[X > 0.2] = 1 - P[X \leq 0.2] \\ = 1 - F_X(0.2) = 1 - 0.7 = 0.3$$

$$\textcircled{6} P[X \leq -\infty] = F_X(-\infty) = 0$$

$$\textcircled{7} P[X \leq \infty] = F_X(\infty) = 1$$



Cumulative density function [cdf]
 $F_X(x)$

Probability density function [pdf]

$$f_X(x) = \frac{dF_X(x)}{dx}$$

pdf ← cdf

$$F_X(x) = \int_{-\infty}^x f_X(u) du$$

Subject: / /

$$P[a \leq x \leq b] = \int_a^b f_x(x) dx$$

$\hookrightarrow$ pdf

$$P[X \leq b] - P[X \leq a]$$

$\hookrightarrow \int_{-\infty}^b f_x(x) dx$

$$\int_{-\infty}^{\infty} f_x(x) dx = 1$$

$$P[a \leq x \leq b] = P[X \leq b] - P[X \leq a]$$

$$= \int_{-\infty}^b f_x(x) dx - \int_{-\infty}^a f_x(x) dx$$

$$= \int_a^b f_x(x) dx$$