Subject: .... / /

$$P[a \le X \le b] = \int_a^b f_X(x)\,dx$$

$\hookrightarrow$ pdf

$$P[X \le b] - P[X \le a]$$

$$\int_{-\infty}^{b} f_X(x)\,dx$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$P[a \le X \le b] = P[X \le b] - P[X \le a]$$

$$= \int_{-\infty}^{b} f_X(x)\,dx - \int_{-\infty}^{a} f_X(x)\,dx$$

$$= \int_a^b f_X(x)\,dx$$

Ch 2: Random variables [r.v]

CDF : Cumulative Density function

$\hookrightarrow$ real numbers

$F_X(x) = pr\{X \le x\}$

$\hookrightarrow$ random variable

probability density function [pdf]

$f_X(x)$: pdf is normally used to model a given experiments

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$f_X(x) = \frac{d F_X(x)}{dx}$, $F_X(x) = \int_{-\infty}^{x} f_u(u)\, du$

Ex/ A pdf is given by

$$f_X(x) = \begin{cases} kx & 0 < x < 2 \\ 0 & \text{o.w} \end{cases}$$

1) Determine the value of k?

2) Find and sketch the CDF, $F_X(x)$?

3) Find $P[0 < X < 1]$?

1) I need to find the value of k such that the pdf is aprobablistic function

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_0^2 kx\, dx = 1 \Rightarrow k \left. \frac{x^2}{2} \right|_{x=0}^{x=2} \Rightarrow k[2] = 1$$

$$k = \frac{1}{2}$$

2) $F_X(x) = \int_{-\infty}^{x} f_u(u)\, du$

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$$f_X(x) = \begin{cases} 0 & x < 0 \\ \int_0^x \frac{1}{2} u \, du & 0 < x < 2 \\ \int_0^2 \frac{1}{2} u \, du & x \geq 2 \end{cases}$$

قيمة ال x
غير معروفة

$$\int_0^x \frac{1}{2} u \, du = \frac{1}{2} \cdot \frac{u^2}{2} \Big|_0^x = \frac{1}{4} x^2$$

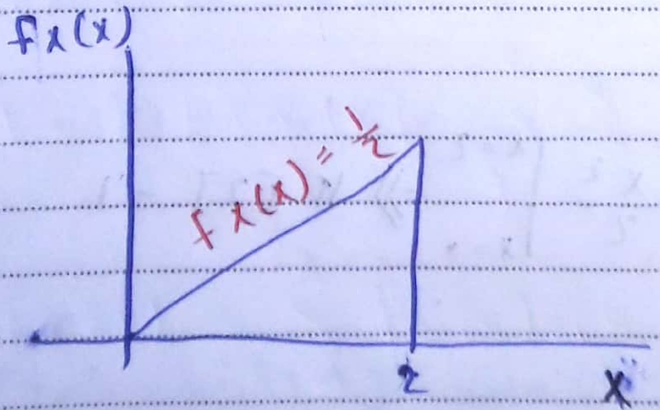
$$\int_0^2 \frac{1}{2} u \, du = \frac{1}{4} u^2 \Big|_0^2 = 1$$

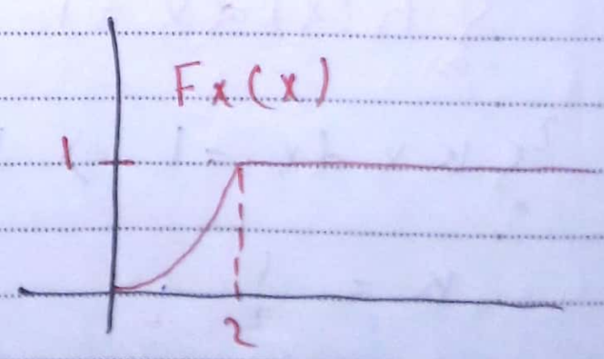
إذا ما طلع (1) معناتو
أنو قيمة ال x ~~بتساوي~~
يلي طلعتها غلط

$F_X(\infty) = 1 \qquad F_X(-\infty) = 0$

$$= \begin{cases} 0 & x < 0 \\ \frac{1}{4} x^2 & 0 < x < 2 \\ 1 & x \geq 2 \end{cases}$$

3) $P[X \leq 2] = F_X(2) = 1$





4) $P[X > 1] = 1 - P[X \leq 1] = 1 - F_X(1)$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

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5) $P[0 < x < 1]$

$= P[x < 1] - P[x < 0]$

$= F_x(1) - F_x(0) = \frac{1}{4} - 0 = \frac{1}{4}$

(P) A pdf is given by

$$f_x(x) = \begin{cases} c x^2 & 0 < x < 3 \\ 0 & o.w \end{cases}$$

1) find the value of c?
2) find the cDf $F_x(x)$?
3) draw the cDf?
4) Find $P[x < 1]$?
5) $P[x < 2]$?
6) $P[x > 1.5]$?
7) $P[2 < x < 4]$?

5) $P[0 < x < 1] = \int_0^1 \frac{1}{2} x \, dx$ حل المثال السابق بطريقة ثانية

$P[x < 2] = \int_0^2 \frac{1}{2} x \, dx$.

$P[x > 1] = 1 - P[x \leq 1] = 1 - \int_0^1 \frac{1}{2} x \, dx$

Ex / A pdf is given by

$$f_x(x) = \begin{cases} 0 & o.w \\ \frac{x-3}{25} & 3 \leq x < 8 \\ \frac{1}{5} - \frac{x-8}{25} & 8 \leq x < 13 \end{cases}$$

Subject:.....................   / /

1) Find $P[X \le 5]$?

$$P[X \le 5] = \int_3^5 \left(\frac{x-3}{25}\right) dx = \frac{1}{25}\left[\frac{x^2}{2} - 3x\right]\Big|_3^5$$

$$= \frac{1}{25}\left[\frac{25}{2} - 15\right] - \left(\frac{9}{2} - 9\right)$$

$$= \frac{1}{25}\left[\frac{-5}{2} - \frac{-9}{2}\right] = \frac{1}{25}[2] = \frac{2}{25}$$

2) $P[4.5 \le X \le 6.7]$?

$$= \int_{4.5}^{6.7} \frac{x-3}{25} dx = 0.2288$$

3) $P[6 < X < 10]$?

$$= \int_6^8 \left(\frac{x-3}{25}\right) dx + \int_8^{10} \left[\frac{1}{5} - \frac{(x-8)}{25}\right] dx$$

4) $P[X > 10]$?

$$1 - P[X \le 10] = \int_3^8 \left(\frac{x-3}{25}\right) dx + \int_8^{10} \left[\frac{1}{5} - \frac{(x-8)}{25}\right] dx$$

$$F_X(x) = \int_{-\infty}^{x} f_u(u)\, du \qquad \Big|_{-\infty}^{\infty}$$

$$F_X(x) = \begin{cases} 0 & x < 3 \\ \int_3^x \frac{u-3}{25} du & 3 \le x < 8 \end{cases}$$

S T A R S Notebook

Subject:...................... / /

$$F_X(x)\begin{cases} \int_3^8 \frac{u-3}{25}\,du + \int_8^x \left[\frac{1}{5} - \frac{u-8}{25}\right] du & 8 \le x < 13 \\ \underbrace{\int_3^8 \frac{u-3}{25}\,du + \int_8^{13} \left[\frac{1}{5} - \frac{u-8}{25}\right] du}_{=1} & x \geq 13 \end{cases}$$

(1) $\int_3^x \frac{u-3}{25}\,du = \frac{1}{25}\left[\frac{u^2}{2} - 3u \Big|_3^x\right]$

$= \frac{1}{25}\left[\left(\frac{x^2}{2} - 3x\right) - \underbrace{\left(\frac{9}{2} - 9\right)}_{-9/2}\right]$

$= \frac{1}{25}\left[\frac{x^2}{2} - 3x + \frac{9}{2}\right]$

(2) الجزء الأول

$\int_3^8 \frac{u-3}{25}\,du = \frac{1}{25}\left[\frac{u^2}{2} - 3u \Big|_3^8\right]$

$= \frac{1}{25}\left[(32 - 24) - \underbrace{\left(\frac{9}{2} - 9\right)}_{-9/2}\right]$

$= \frac{1}{25}\left[8 + 4.5\right]$

$= \frac{12.5}{25} = 0.5$

(2) الجزء الثاني

$\int_8^x \left[\frac{1}{5} - \frac{u-8}{25}\right] du$

$= \frac{1}{5}[x - 8] - \frac{1}{25}\left[\frac{u^2}{2} - 8u \Big|_8^x\right]$

$= \frac{1}{5}[x - 8] - \frac{1}{25}\left[\frac{x^2}{2} - 8x - \underbrace{(32 - 64)}_{-32}\right]$

$$= \frac{1}{25}\left[5x - 40\right] - \frac{1}{25}\left[\frac{x^2}{2} - 8x + 32\right]$$

$$= \frac{1}{25}\left[13x - 72 - \frac{x^2}{2}\right]$$

$$= \begin{cases} 0 & x < 3 \\ \frac{1}{25}\left[\frac{x^2}{2} - 3x + \frac{1}{2}\right] & 3 \le x < 8 \\ 0.5 + \frac{1}{25}\left[13x - 72 - \frac{x^2}{2}\right] & 8 \le x < 13 \\ 1 & x > 13 \end{cases}$$

الى هنا أمتحان الفيرست

* للتاكد من صحة الحل يجب أن نعوض الـ 8 في الجزئين لانه اقتران متصل